SUPERSONIC FLOW OVER A TRIANGULAR WING

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The nature of the flow in a three-dimensional boundary layer on a triangular wing in hypersonic flow of a gas at zero angle of attack can vary widely. The parameters governing the type of flow and its special features are the sweepback angle of the lading edge, the surface temperature, the interaction parameter, and other characteristics.

Symmetric flow over a thin triangular wing in the strong viscous interaction regime was examined in [1], but the similarity solution obtained does not satisfy the condition of no flow through the plane of symmetry of the wing.

In further investigations [2, 3] for flows on noncooled bodies in the strong viscous interaction regime it was shown that the solution near the leading edge is not unique, and this allows us to satisfy boundary conditions at the trailing edge of the body or in the plane of symmetry of the triangular wing. There are considerable difficulties associated with constructing a global solution, due to the need to allow for the flow structure near the wing plane of symmetry. Some special kinds of possible local flows were pointed out and obtained in [4, 5].

In [6, 7] the authors obtained global solutions of the equations of the three-dimensional boundary layer in the strong viscous interaction regime, under the assumption that these equations are valid over the entire noncooled wing.

Certain questions associated with transmission of perturbations on cold bodies and in the wake were considered in [8]. The present paper studies special features associated with propagation of perturbations in supersonic regimes of flow over cold triangular wings, and numerical solutions have been obtained.

1. We consider flow of a hypersonic viscous gas over a triangular semi-infinite wing (Fig. 1). We introduce a rectangular coordinate system with axes xl and yl directed along the normal to one of the edges and along the edge, and the $y\tau l$ axis directed along the normal to the wing surface. We consider the regime of strong interaction of the flow in the boundary layer with the external inviscid hypersonic stream [9] for the case of an asymptotically small ratio of surface temperature to stagnation temperature. The dimensionless boundary layer thickness is on the order of $\delta^*/l \sim \tau = \mathrm{Re}_0^{-1/4}$, where $\mathrm{Re}_0 = \rho_\infty U_\infty l/\mu_0$ is the Reynolds number reckoned from the values of gas density and velocity in the unperturbed stream, the viscosity at the stagnation temperature ($\mu_0 = \mu(T = T_0)$), and the characteristic length l, which drops out from the finite results for the similarity problem. For the components of the velocity vector in the directions of the x, y, and z axes, the pressure, the stagnation enthalpy, and the viscosity we introduce the following notation:

$$U_\infty\bar{u},\ \tau U_\infty\bar{v},\ U_\infty\bar{w},\ \tau^2\rho_\infty\bar{\rho},\ \tau^2\rho_\infty U_\infty^2\bar{p},\ (U_\infty^2/2)\bar{g},\ \mu_0\bar{\mu}.$$

The equations of the three-dimensional boundary layer and the boundary conditions, after substitution of the variables

$$\lambda = x^{-1/4}\int_0^y \bar{\rho}\,dy,\ \zeta = \omega_0^{-1}\,\mathrm{arctg}\,(x/z),\ \bar{\delta} = x^{1/4}\int_0^\infty \bar{\rho}^{-1}d\lambda, \tag{1.1}$$

$$\bar{u} = \frac{\partial f}{\partial \lambda},\ \bar{w} = \frac{\partial \varphi}{\partial \lambda},\ \bar{\rho} = x^{-1/2}\rho,\ \bar{g} = g,\ \bar{p} = x^{-1/2}p,\ \bar{\delta} = x^{3/4}\delta$$

(ω_0 is the angle between the z axis and the direction of the incident flow), are reduced to the form

$$(Nf'')' + \frac{1}{4}ff'' + \frac{\gamma-1}{4\gamma}\left(1 - \frac{\dot{p}\sin 2\omega}{p\omega_0}\right)(g - f'^2 - \varphi'^2) = \tag{1.2}$$

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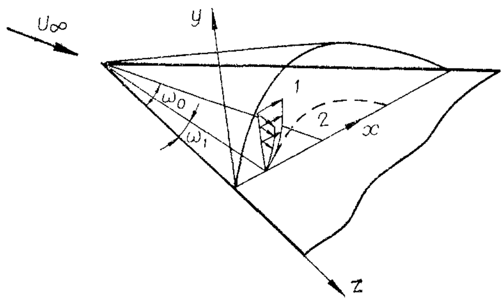

Fig. 1

$$= \frac{\sin 2\omega}{2\omega_0}\left(f'\dot{f}' - f''\dot{f}\right) - \frac{\sin^2\omega}{\omega_0}\left(\varphi'\dot{f}' - f''\dot{\varphi}\right),$$

$$(N\varphi'')' + \frac{1}{4}f\varphi'' + \frac{\gamma-1}{2\gamma}\frac{\dot{p}\sin^2\omega}{p\omega_0}(g - f'^2 - \varphi'^2) =$$

$$= \frac{\sin 2\omega}{2\omega_0}\left(f'\dot{\varphi}' - \varphi''\dot{f}\right) - \frac{\sin^2\omega}{\omega_0}\left(\varphi'\dot{\varphi}' - \dot{\varphi}\varphi''\right),$$

$$\left(N\sigma^{-1}g'\right)' + \frac{1}{4}fg' + N\left(1 - \frac{1}{\sigma}\right)(f'^2 + \varphi'^2) =$$

$$= \frac{\sin 2\omega}{2\omega_0}\left(f'\dot{g} - g'\dot{f}\right) - \frac{\sin^2\omega}{\omega_0}\left(\varphi'\dot{g} - g'\dot{\varphi}\right),$$

$$N = \frac{2\gamma}{\gamma-1}p\,(g - f'^2 - \varphi'^2)^{n-1},\ \ \omega = \omega_0\zeta,\ \ \delta = \frac{\gamma-1}{2\gamma p}\int\limits_0^\infty (g - f'^2 - \varphi'^2)\,d\lambda,$$

$$f_w = \varphi_w = f_w' = \varphi_w' = g_w = 0,\ \ f_e' = \sin\omega_0, \varphi_e' = \cos\omega_0, g_e = 1,$$

$$p = \frac{\gamma+1}{2}\left[\frac{3}{4}\delta\sin\omega_0 + \frac{1}{\omega_0}\frac{d\delta}{d\zeta}\left(\frac{1}{2}\sin(2\omega)\sin\omega_0 - \sin^2\omega\cos\omega_0\right)\right]^2.$$

Below we consider the case of linear dependence of the viscosity on temperature ($n = 1$) and $\sigma = 1$.

The nature of the flow around a cold triangular wing depends on the regime of interaction of the boundary layer with the external hypersonic stream [8]. In the subsonic regime the perturbations can be propagated upstream, in the case considered, from the plane of symmetry to the leading edge. In choosing a unique solution one must take into account the additional boundary condition, e.g., the value of the pressure on some surface ζ = const. In the supersonic flow region the perturbations cannot be propagated upstream and the flow is described by the similarity solution of the system of equations (1.2). On a cold wing the variation of the displacement thickness is generated by the main part of the boundary layer, and this variation depends linearly on the pressure perturbations [8]. The interaction regime is determined by the sign of the derivative $d\delta/dp$, for $d\delta/dp < 0$; we have the supersonic regime, and for $d\delta/dp > 0$ we have the subsonic regime.

For reduced wing sweepback angle $\chi = \pi/2 - \omega_0$, the quality $d\delta/dp|_{\zeta=0}$ decreases monotonically, going to zero for a certain critical value sweepback angle [8]. Thus, in the boundary layer on a triangular wing for $\chi^* = \pi/2 - \omega^*$ there are regions of supersonic and subsonic flow, and the transition from one type of flow to the other occurs at some value $\omega_0 > \omega^*$ [8]. The flow in the region between the leading edge and the surface $\omega^* < \omega_1 < \omega_0$ [8] is described by the similarity solution of the system of equations (1.2). In constructing a solution in the region between the surface $\omega = \omega_1$ and the wing plane of symmetry we must take account of the influence of transmission of perturbations. Viscosity forces do not influence the perturbation flow in the boundary layer for $\lambda = O(1)$ near the surface $\omega = \omega_1$. Analysis of the system of equations describing this kind of flow shows that the system can be integrated once with respect to ζ, whence it follows that the perturbations of the functions f and q are proportional to the pressure perturbation $p_1(\zeta_1)$. We represent the solution to the right of the surface $\omega = \omega_1$ in the form of expansions for the stream functions:

$$p = p_0 + p_1(\zeta_1) + \ldots, \quad \delta = \delta_0 + \frac{p_1(\zeta_1)}{p_0}\delta_1 + \ldots,$$

$$f = f_0(\lambda) + \frac{p_1(\zeta_1)}{p_0} f_1(\lambda) + \ldots, \quad \varphi = \varphi_0(\lambda) + \frac{p_1(\zeta_1)}{p_0}\varphi_1(\lambda) + \ldots, \tag{1.3}$$

$$g = g_0(\lambda) + \frac{p_1(\zeta_1)}{p_0} g_1(\lambda) + \ldots, \quad \zeta_1 = \zeta - \frac{\omega_1}{\omega_0},$$

where the functions with the subscript 0 correspond to the similarity solution. Substitution of Eq. (1.3) into the system of equations (1.2) gives a linear system of equations for the first approximation, for which the solution was obtained in [8]. We note that for $\lambda = O(1)$ this solution does not depend on the form of the function $p_1(\zeta_1)$. The expression for the variation of the displacement thickness generated in the region $\lambda = O(1)$ has the form

$$\delta_1 = \frac{\gamma - 1}{2\gamma p_0}\Delta p\left[\frac{\gamma - 1}{2}\int_0^\infty \left(\frac{g_0 - f_0'^2 - \varphi_0'^2}{f_0' \cos\omega_1 - \varphi_0' \sin\omega_1}\right)^2 d\lambda - \int_0^\infty (g_0 - f_0'^2 - \varphi_0'^2)\, d\lambda\right], \tag{1.4}$$

where $\Delta p = p_1/p_0$.

The derivative $d\delta_1/d\Delta p$ depends linearly on the coordinate ζ. By differentiating Eq. (1.4) with respect to ζ we obtain the expression

$$\frac{d\delta_1}{d\Delta p} = \frac{(\gamma - 1)^2}{4\gamma p_0}\omega_0 J_1 \zeta_1, \quad J_1 = 2\int_0^\infty \frac{(g_0 - f_0'^2 - \varphi_0'^2)^2 (f_0' \sin\omega_1 + \varphi_0' \cos\omega_1)}{(f_0' \cos\omega_1 - \varphi_0' \sin\omega_1)^3} d\lambda. \tag{1.5}$$

The solution for the first approximation is not uniformly accurate, since we did not account for viscous forces. To satisfy the boundary conditions assigned on the wing surface we must consider region 2, in which the influence of viscous and inertial forces is the same in the first approximation. The wall region 2 induces a change of the displacement thickness Δ_1. Estimates for the thickness of region 2 and for the scales of the functions in this region are obtained by equating in the system of equations the orders of terms accounting for the influence of viscous forces and inertial forces. Finally, the requirement of matching the solutions in regions 1 and 2 leads to equality of the orders of the pressure gradient and the inertial terms, which allows us, for a known thickness of region 2, to find the scales of the perturbations functions f, φ, and g. An expression for the variation of the displacement thickness generated in region 2 has the form

$$\Delta_1 = c_1 p_1 (p_1/\dot{p}_1)^{1/3} \ (c_1 \text{ constant}). \tag{1.6}$$

To determine the function $p_1(\zeta_1)$ we must use the interaction condition. The assumption that the main contribution to variation of the displacement thickness is formed in region 1 leads, allowing for the interaction condition, to a function of the form $p_1 = c\zeta_1^\alpha$. The flow in region 2 then induces the change of displacement thickness $\Delta_1 \simeq \zeta_1^{1/3} p_1$, which is greater by an order of magnitude than δ_1 for all allowable values of α. The assumption that the main contribution to the displacement thickness comes from region 2 also does not lead to a self-matching scheme. In this case it turns out that the induced pressure perturbation is larger than the original perturbation.

Analysis of the interaction condition shows that the solution of Eq. (1.3) exists if the total change of the displacement thickness in the first approximation is zero:

$$\Delta\delta = \delta_1 + \Delta_1 = 0. \tag{1.7}$$

A similar kind of interaction was described in [10], which studied flow in the vicinity of the trailing edge of a flat plate, and in [11], which investigated flow over small roughnesses on the floor of a laminar boundary layer. An expression for the function $p_1(\zeta_1)$ can be obtained from Eqs. (1.5) and (1.6), and has the form

$$p_1(\zeta_1) = c\exp(-\alpha/\zeta_1^2). \tag{1.8}$$

The total change of displacement thickness can be obtained from Eq. (1.2):

$$\Delta\delta = \frac{\gamma - 1}{2\gamma p_0}\Delta p\left[\int_0^\infty (g_1 - 2f_0'f_1' - 2\varphi_0'\varphi_1' - g_0 + f_0'^2 + \varphi_0'^2)\, d\lambda + \zeta_1\int_0^\infty (g_2 - g_1(0))\, d\eta\right] + O(\zeta_1^2 p_1/p_0), \quad \eta = \frac{\lambda}{\zeta_1^{1/2}}. \tag{1.9}$$

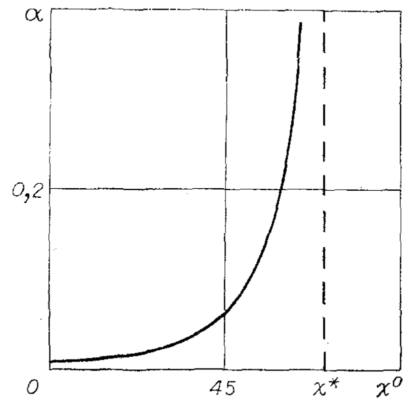

Fig. 2

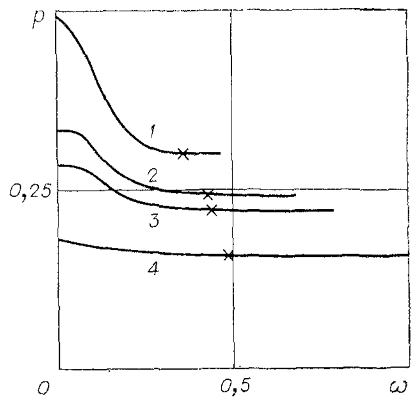

Fig. 3

The first integral on the right of Eq. (1.9) is the contribution of region 1, and, according to Eq. (1.5), can be written in the form $\delta_1 = (\gamma - 1)^2 \Delta p \zeta_0 J_1 \zeta_1 / 4\gamma p_0$. The second integral in Eq. (1.9) is the contribution of the wall region to the change of the displacement thickness, and to find this integral one must obtain a solution for region 2.

In region 2 we introduce the representation of the function

$$\begin{aligned} f &= a\eta^2\zeta_1/2 + \frac{c}{p_0}\zeta_1^{1/2}\exp\left(-\alpha/\zeta_1^2\right) f_2(\eta) + \ldots, \\ \varphi &= b\eta^2\zeta_1/2 + \frac{c}{p_0}\zeta_1^{1/2}\exp\left(-\alpha/\zeta_1^2\right)\varphi_2(\eta) + \ldots, \\ g &= d\eta^2\zeta_1^{1/2} + \frac{c}{p_0}\exp\left(-\alpha/\zeta_1^2\right) g_2(\eta) + \ldots, \\ p &= p_0 + c\exp\left(-\alpha/\zeta_1^2\right) + \ldots, \quad \eta = O(1), \end{aligned} \tag{1.10}$$

where c is an arbitrary constant. The first terms of the expansions for the functions f, φ, and g are asymptotic representations of the functions f_0, φ_0, and g_0 for $\lambda \to 0$. The parameters a, b, and d are determined from the similarity solution. Substitution of Eq. (1.10) in the system of equations (1.2) leads, after a number of transformations, to the following system of equations for the first approximation:

$$z_1''' - \eta_1 = \eta_1 z_1', \quad z_2''' - \eta_1 = \eta_1 z_2' - z_2, \quad z_3'' = \eta_1 z_3 - z_2,$$

$$z_1(0) = z_1'(0) = z_2(0) = z_2'(0) = z_3(0) = 0,$$

$$z_1'(\infty) = -1 + o(1), \quad z_2'(\infty) = -\ln\eta_1 + O(1), \quad z_3'(\infty) = -\ln\eta_1 + O(1),$$

where

$$z_1 = \frac{2^{4/3}\alpha^{1/3}\gamma\omega_0 T^{4/3}(bf_2 - a\varphi_2)}{d(\gamma-1)\sin\omega_1(b\cos\omega_1 + a\sin\omega_1)};$$

$$z_2 = \frac{2^{4/3}\alpha^{1/3}\gamma\omega_0 T^{4/3}(f_2\cos\omega_1 - \varphi_2\sin\omega_1)}{d(\gamma-1)\sin\omega_1};$$

$$z_3 = \frac{2\omega_0^2 T^2\gamma g_2}{d(\gamma-1)\sin^2\omega_1}; \quad \eta = \eta_1 T^{-1/3} 2^{-1/3}\alpha^{-1/3}; \quad T = \frac{\sin\omega_1}{\omega_0}(a\cos\omega_1 - b\sin\omega_1).$$

Taking account of Eqs. (1.7) and (1.9), the solution of the system of equations describing the solution in region 2 leads to an expression for the eigenvalue α

$$\alpha = \left(d^2 J_2 \sin^2\omega_1/\gamma\omega_0^3\right)^3/2, \quad J_2 = \int_0^\infty (z_3(\eta_1) - z_3(\infty))\, d\eta.$$

The dependence $\alpha(\chi)$ is shown in Fig. 2. It can be seen that as the sweepback angle $\chi = \pi/2 - \omega_0$ increases, the eigenvalue $\alpha(\chi)$ increases monotonically and tends to infinity for $\chi \to \chi^* = \pi/2 - \omega^*$. The increase of the eigenvalue α is associated with a decrease of the degree of transmission of perturbations upstream. This kind of change in the nature of transmission of perturbations can be explained by the fact that the change of the displacement thickness of region 2 depends inversely on powers of both the eigenvalue α_1 and the parameter ω_1.

The condition (1.7) requires conservation of the order of the quantity Δ_1, and therefore, as we reduce the distance from the leading edge to the plane where the perturbed flow begins, the eigenvalue must increase.

The physical explanation of the increase of the intensity of transmission of perturbations (decrease of α) with reduced wing sweepback angle χ is connected with the fact that with increase of ω_0 we have an increased length (with respect to ζ) of the supersonic flow region, and therefore the gradients of the stream functions are reduced, the velocity profiles become less full, and we increase the thickness of the layer of subsonic jets.

For $\omega_0 \to \omega^*$ the functions $\sin^2\omega$ and $\sin 2\omega$, appearing in the system of equations (1.2), can be approximated by the first terms of a series expansion, since $\omega = \omega_1 + \omega_0\zeta = o(1)$. Then to evaluate the thickness of region 2 we have

$$\lambda_2 \sim \left[\frac{p_1}{(\omega_1 + \omega_0\zeta_1)\dot{p}_1}\right]^{1/3}.$$

Correspondingly, the change of displacement thickness generated in region 2 is determined as follows:

$$\Delta_1 = Ap_1\left[\frac{p_1}{(\omega_1 + \omega_0\zeta_1)\dot{p}_1}\right]^{1/3}, \quad A = \left(\frac{\gamma - 1}{2\gamma}\right)^2 \frac{d^2\omega_0^{1/3}J_2}{p_0 a^{7/3}} < 0. \tag{1.11}$$

Condition (1.7), allowing for the expression for Δ_1, takes the form

$$B\zeta_1 p_1 + Ap_1\left[\frac{p_1}{(\omega_1 + \omega_0\zeta_1)\dot{p}_1}\right]^{1/3} = 0, \quad B = \frac{(\gamma - 1)^2}{4\gamma p_0}\omega_0 J_1. \tag{1.12}$$

The solution of the differential equation (1.12) is

$$p_1 = c\exp\left\{\frac{A^3}{\omega_0 B^3}\left(\frac{\omega_0}{\omega_1}\right)^3\left[\ln\frac{\omega_0\zeta_1}{\omega_1 + \omega_0\zeta_1} + \frac{\omega_1}{\omega_0\zeta_1} - \frac{\omega_1^2}{2\omega_0^2\zeta_1^2}\right]\right\}. \tag{1.13}$$

For $\omega_1 = O(1)$ the expression (1.12) coincides in its main term with Eq. (1.8). For $\omega_1 \to 0$ and for a small but fixed value of ζ_1 the right side of Eq. (1.13), after expansion in a series with respect to the small parameter $\omega_1/(\omega_0\zeta_1)$, reduces to the function

$$p_1 = c\exp\left(\frac{A^3}{3B^3\omega_0\zeta_1^3}\right). \tag{1.14}$$

For $\omega_0 \to \omega^*$ ($\omega_0 - \omega^* < 0$) the change of the displacement thickness in region 1 can be represented in the form $\delta_1 = B(\zeta_1 - 1 + \omega^*/\omega_0)p_1$. An estimate for the change of the displacement thickness of layer 2 is obtained from Eq. (1.11), if we put $\omega_1 = 0$ there. Correspondingly, condition (1.7) has the form

$$B\left(\zeta_1 - 1 + \frac{\omega^*}{\omega_0}\right)p_1 + Ap_1\left(\frac{p_1}{\omega_0\zeta_1\dot{p}_1}\right)^{1/3} = 0. \tag{1.15}$$

The solution of Eq. (1.15) will then be

$$p_1 = c\exp\left\{\frac{A^3}{B^3(\omega^* - \omega_0)^3}\left[\ln\frac{\omega_0\zeta_1}{\omega^* - \omega_0 + \omega_0\zeta_1} + \frac{\omega^* - \omega_0}{\omega^* - \omega_0 + \omega_0\zeta_1} + \frac{1}{2}\left(\frac{\omega^* - \omega_0}{\omega^* - \omega_0 + \omega_0\zeta_1}\right)^2\right]\right\}. \tag{1.16}$$

For finite values of the parameter $\omega^* - \omega_0$ we obtain a power series eigenfunction of the type $p = c\zeta_1^{\alpha}$, which agrees with the results of [8]. The other limiting transition $\omega_0 \to \omega^*$ leads to the function (1.14). Thus, we have achieved a continuous transition of the coordinate expansions near the leading edge for the subsonic flow regime to expansions describing the beginning of interaction in the flow containing subsonic and supersonic regions.

2. Using the method described in [7] we have obtained a numerical solution of the system of equations of the three-dimensional boundary layer on a triangular wing in the regime of strong viscous interaction. It should be noted that in the calculations we used a system of boundary layer equations in a coordinate system fixed to the wing symmetry axis. For reasons of brevity this system is not presented here, and the results of the calculations are

represented in the variables of Eq. (1.1). In the numerical integration of the system of equations the region in the vicinity of $\omega = \omega_1$ was not treated specially, and the boundary problem was solved from one leading edge to the other.

In the calculations it was assumed that $\sigma = 1$, $n = 1$, $\gamma = 1.4$, $g_w = 0$. In Fig. 3, curves 1-4 show the pressure distribution with respect to the coordinate ω for different lengths of wing $S_0 = \tan\omega_0 = 0.5$; 0.8; 1; 2. The crosses denote values of the coordinate ω_1 for which there was transition from the supersonic to the subsonic regime, in accordance with the expression going to zero for the change of the displacement thickness of Eq. (1.4). As can be seen from the results of the numerical solution of the boundary problem presented, the departure from the similarity solutions, i.e., the transition from supersonic to subsonic flow, takes place in accordance with the values of ω_1 given by Eq. (1.4). Thus, ignoring the fine structure for $\omega = \omega_1$ in solving the global problem does not lead to significant errors in determining the pressure distribution.

The flows studied in this work, with regions of subsonic and supersonic interaction, were set up for quite smooth boundary conditions, assigned downstream. As was mentioned in [8], one may find other types of flows with supersonic and subsonic regions, where the transition from one region to the other occurs in small distances because of large perturbations assigned downstream. Further analytical and computational investigations are required to analyze such flows.

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